SOME NEW RESULTS ON COUPLED OR MEANDER MICROSTRIP LINES BY APPLICATION OF A MATRIX THEORY

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Abstract

Knowing the elements of capacitance matrices for coupled microstrip lines, we are able to obtain the characteristics of coupled or meander lines by application of a matrix theory. The elements of the capacitances matrices previously computed from geometrical dimensions of the lines can now be obtained directly by analytical formulas in a large domain of values of ω, s, h and ϵ_r.

Introduction

The determination of all the parameters of a shielded or non-shielded microstrip coupler, in symmetrical position, is the subject of many papers. Some problems have not been solved, like the matching conditions, and the calculation of parameters taking into account the divergence between the velocities of the various eigen modes of propagation. This paper is based on Ref. 1.

Matrix formulation

Let us consider a set of n shielded microstrip coupled lines, supported by a dielectric substrate (Fig. 1). Suppose that the principal mode of propagation is a quasi T.E.M. mode. Let oz be the axis of propagation parallel to the lines. When potentials $V_1, V_2, \dots V_n$ are applied to the ports z = 0, currents $I_1, I_2, \dots I_n$ flow in the lines. We can write the equations of propagation in a matrix formalism where $\overline{V}(z)$ and $\overline{I}(z)$ are column vectors.

$$\begin{cases} \left(\frac{d^2}{dz^2} U + \omega^2 S M \right) \overline{I}(z) = 0 \\ \left(\frac{d^2}{dz^2} U + \omega^2 M S \right) \overline{V}(z) = 0 \end{cases} \quad (1)$$

ω is the frequency of the incident wave. In these expressions are introduced the matrices (S), (M) and (U). The latter one is the unit matrix. The first ones are important characteristic matrices: (S) is the matrix of self and mutual capacitances, its elements can be easily computed from an accelerated finite differences method previously established[2]. (M) is the correspondant matrix of self and mutual admittances. We can also compute all its elements from $(M) = (I/C^2)(S_o)^{-1}$ where C and (S_o) are the velocity of light and the matrix (S) in vacuum. We show[1] that the eigen values of the matrix product (G) = (S) (M) are related to the phase velocities of the modes of propagation. Having n lines, there are n eigen values α_i, and so n velocities of propagation v_i since $v_i = 1/\sqrt{\alpha_i}$. For each α_i, corresponding eigen modes of propagation can be calculated, so it is possible to calculate the values in voltage and current which must be applied respectively on the N input ports to excite our eigen wave travelling through the device. For a two lines symmetrical coupler these voltages or currents excite the well known even and odd modes. Then we can obtain other matrices as the characteristic impedance matrix (Z_c) by:

$$(Z_c)\ (S)\ (Z_c) = (M) \quad (2)$$

as the input impedance matrix (Z_{in}) defined by $\overline{V}(o) = (Z_{in})\ \overline{I}(o)$ and as the well known scattering matrix of the circuit. Then these matrices are also easily computed from the important matrix (S): this computation taking into account the divergence of the velocities of propagation and the frequency.

Matching conditions and couplers

If n coupled lines are loaded by impedance Z_o at each port, then the boundary conditions can be expressed as $\overline{V}(o) = \overline{V}_o(o) - Z_o\overline{I}(o)$ with $V_1(o) = Z_o I_1(o)$ and $\overline{V}(\ell) = Z_o\ \overline{I}(\ell)$. For matching, the load impedance Z_o should be the solution of:

$$\det\left((Z_{in}) - Z_o(D)\right) = 0 \quad (3)$$

Asymmetrical and symmetrical couplers applications

First we give some results on velocities of propagation, characteristic matrix, matching impedance with respect to frequency and asymmetry. The geometrical dimensions of the studied coupler follow: (Fig. 1) A = 32 mm; B = 3.7 mm; W = 2.2 mm; S = 0.8 mm. All the studied couplers are supported by teflon (ϵ_r = 2.65) as dielectric substrate. The length of the lines are calculated in such a way that the coupling factor is maxima at 1.5 GHz.

Capacitances and inductances

The upper theory has given the matrices of capacitances (S) and (M):

K/K'=1
- Self capacitances: $C_1 = 1.046\ 10^{-10}$F
  - $C_2 = 1.046\ 10^{-10}$F
- Mutual capacitances: $S_{12} = -0.737\ 10^{-10}$F
  - $S_{21} = -0.737\ 10^{-10}$F

K/K'=0.29
- Self capacitances: $C_1 = 1.247\ 10^{-10}$F
  - $C_2 = 9.99\ 10^{-11}$F
- Mutual capacitances: $S_{12} = -0.720\ 10^{-10}$F
  - $S_{21} = -0.720\ 10^{-10}$F

Likewise, we have the matrices of inductances (M):

K/K'=1
- Self inductances: $L_1 = 2.82\ 10^{-7}$H
  - $L_2 = 2.82\ 10^{-7}$H
- Mutual inductances: $M_{12} = 1.466\ 10^{-7}$H
  - $M_{21} = 1.466\ 10^{-7}$H

K/K'=0.29
- Self inductances: $L_1 = 2.48\ 10^{-7}$H
  - $L_2 = 1.902\ 10^{-7}$H
- Mutual inductances: $M_{12} = 1.052\ 10^{-7}$H
  - $M_{21} = 1.052\ 10^{-7}$H

Velocities

The phase velocities are now calculated from these matrices. We obtain:

ε_r	K/K'	v_e m/s	v_o m/s
1	1	3.10^8	3.10^8
2.65	1	$2.74\ 10^8$	$2.03\ 10^8$
	0.8	$2.75\ 10^8$	$2.03\ 10^8$
	0.29	$2.76\ 10^8$	$2.04\ 10^8$

Meander lines application

Eigen values for three lines

It is not right to consider one wave number β instead of the three possible values β_i before having calculated and compared them. We have calculated the α_i for various configurations of three lines of equal width regularly spaced using an accelerated over-relaxation method to determine $(S)^1$. We recall that $\beta i = \omega\sqrt{\alpha_i}$. For instance we obtain $\alpha_I = 0.3220 \times 10^{-16}$, $\alpha_{II} = 0.5061 \times 10^{-16}$, $\alpha_{III} = 0.5590 \times 10^{-16}$. In a first approach we can use a mean value α of the α_i, with the corresponding value $\gamma = j\beta$

Parameters of a meander line with N-1 meanders

As long as the previous approximation remains valid for a delay line having N-1 meanders corresponding to N coupled lines we obtain:

$$\begin{bmatrix} \bar{V}(0) \\ \bar{V}(\ell) \end{bmatrix} = \begin{bmatrix} a(Z_c) & b(Z_c) \\ -b(Z_c) & -a(Z_c) \end{bmatrix} \begin{bmatrix} \bar{I}(0) \\ \bar{I}(\ell) \end{bmatrix} \quad (4)$$

with $a = \coth\gamma\ell$, $b = -(1/\mathrm{Sh}\gamma\ell)$. Adjoining limit conditions between the variables, the 2 x N port system reduces to a two port system having an impedance matrix:

$$(Z) = \begin{bmatrix} A & B \\ B & A \end{bmatrix} \quad (5)$$

Then we have the scattering matrix:

$$\Sigma = \Big((Z) - Z_o(U)\Big)\Big((Z) + Z_o(U)\Big)^{-1}$$

$$\Sigma = \begin{bmatrix} s_{II} & s_{IO} \\ s_{IO} & s_{II} \end{bmatrix} \quad \begin{pmatrix} I : \text{input} \\ 0 : \text{output} \end{pmatrix}$$

and we obtain a matching condition for a delay line, $Z_o^2 = A^2 - B^2$. The useful parameters : phase delay and group delay, are obtained from the amplitude and phase of s_{IO}, knowing that :

$$s_{IO} = \frac{1}{A/B + (A/B^2 - 1)^{\frac{1}{2}}}$$

Algorithm of calculation for an example with two meanders (three lines)

Here the limit conditions are:

$$\begin{cases} V_2(\ell) = V_1(\ell) \\ V_3(0) = V_2(0) \end{cases} \qquad \begin{cases} I_2(\ell) = -I_1(\ell) \\ I_3(0) = -I_2(0) \end{cases} \quad (6)$$

$$(Z_c) = \begin{vmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{12} & Z_{22} & Z_{12} \\ Z_{13} & Z_{12} & Z_{11} \end{vmatrix} .$$

If $\delta_1 = Z_{11} - Z_{12}$; $\delta_2 = Z_{12} - Z_{22}$; $\delta_3 = Z_{13} - Z_{12}$; $\delta_4 = \delta_1 - \delta_2$ $\delta_5 = \delta_2 - \delta_3$

Applying the limit conditions (6) it comes:

$$I_1(\ell) = -\alpha I_1(0) - \beta I_3(\ell); \quad I_2(0) = \beta I_1(0) + \alpha I_3(\ell) \quad (7)$$

$$\alpha = \frac{ab(\delta_1\delta_4 + \delta_3\delta_5)}{a^2\delta_4^2 - b^2\delta_5^2} \qquad \beta = \frac{a^2\delta_3\delta_4 + b^2\delta_1\delta_5}{a^2\delta_4^2 - b^2\delta_5^2}$$

Then the (Z) matrix of the two-port 1,0 - 3,ℓ is obtained applying convenient parts of (4):

$$V_1(0) = (Z_{11}Z_{12}Z_{13})\ (a\bar{I}(0) + b\ \bar{I}(\ell)) \quad (8)$$

$$V_3(\ell) = -(Z_{13}Z_{12}Z_{11})\ (b\bar{I}(0) + a\ \bar{I}(\ell)) \quad (9)$$

Eliminating the intermediate variables $I_1(\ell)$ and $I_2(0)$ from (7), (8), and (9) give:

$$\begin{bmatrix} V_I(0) \\ V_3(\ell) \end{bmatrix} = \begin{bmatrix} A & B \\ B & A \end{bmatrix} \begin{bmatrix} I_1(0) \\ I_3'(\ell) \end{bmatrix} \quad (10)$$

with $I_3'(\ell) = -I_3(\ell)$ (because the orientations)

and : $\begin{cases} A = (Z_{11} - \beta\delta_3)\ a - \alpha\delta_1 \ \text{(b)} \\ -B = -\alpha\ \delta_3 a + (Z_{13} - \beta\delta_1)\ b \end{cases}$

The phase and group delay will be obtained from s_{IO} with :

$$A/B = \frac{(Z_{11} - \beta\delta_3)\ ah\gamma\ell + \alpha\delta_1}{\alpha\ \delta_3 ch\gamma\ell + Z_{13} - \beta\delta_1}$$

The calculations will be then purely numerical, s_{IO} being written :

$$s_{IO} = |s_{IO}| e^{j\phi}$$

the group delay is obtained as : $\tau = -\frac{d\phi}{d\omega}$

The attenuation is given by $|s_{IO}|$

If we consider an "ideal" meander line with lines of length $\lambda/4$ it comes for the matching condition:

$$Z_o^2 = \frac{1}{4}\left[Z_{13}^2 - 2\beta\ \delta_1\ Z_{13} - \delta_1^2\ (\alpha^2 - \beta^2)\right]$$

For a line with three meanders or more the algorithm of calculation will be the same, if it is possible to take the same velocities on the lines without a too important error.

Calculation of the matrix (S)

We have seen that all the parameters of microstrip lines or couplers can be calculated from the matrix (S) for a non-magnetic substrate. Up to now, capacitances were computed by different numerical methods and not very easy to use. So we have to try to obtain analytical expressions for capacitances, impedances, effective dielectric constant, coupling coefficient which only depend on ε_r, w/h and s/h. These expressions are usable for all the positive values of these parameters, but they were obtained from computing results for $1 \leq \varepsilon_r \leq 100 \cdot 0.04 \leq w/h \leq 10$ for single lines and for $0.03 \leq s/h \leq 5$ for coupled lines.

Firstly we have studied the single inhomogeneous microstrip line. Fixing w/h, we have calculated the capacitances with several values of ε_r between 1 and 100.

Plotting the capacitance in terms of ε_r we have obtained a straight line for all w/h. The effect of the parameter ε_r on the capacitance being known, we have fixed ε_r and plotted the slope of straight lines above mentioned in terms of w/h; we have obtained a curve having an oblique assymptotic direction for great values of w/h. This is physically correct because when w/h becomes very great, the capacitance tends to the one of a perfect plane capacitor, the capacitance of which being a linear function of w/h. Having the values of the slope for small values of w/h ($0.04 \leq w/h \leq 1$), we can join this part to the quasi linear one for greater values of w/h ($w/h > 2$) by using a "least squares method." So we have the capacitance for a single inhomogeneous microstrip line in terms of ε_r, w/h and C_o, the capacitance of the line for an homogeneous structure. By a conformal mapping method, it is possible to calculate this capacitance only in terms of ε_o and w/h. Then the final result is that we have a formula giving the capacitance for all the possible configurations.

For the coupled lines, we obtain also a linear variation of the capacitances for odd and even modes with the dielectric constant ε_r. The slopes of these straight lines are on both sides of the slope of the single microstrip line having the same w/h and tends to this one when s/h increases.

Using the self and mutual capacitances which are the sum and difference of the odd and even mode capacitance we have studied their variations with the different geometrical parameters of the line. Fixing firstly s/h, we have obtained the epxressions of self capacitances by reasoning and calculations identical with those used for the single line. Mutual capacitances are also obtained but they are more easily reached by fixing w/h and varying s/h. In each case, if one of the parameters is fixed we obtained the numerical coefficients of the analytical expression of the capacitance in terms of this fixed coefficient. So by varying this one, we obtained a set of values which can be joined one to each other by using a "least square method."

The result is that we give analytical formulas giving self and mutual capacitances or odd and even modes capacitances in terms of ε_r, w/h, s/h and C_{oo} or C_{oe}, the odd and even capacitances for homogeneous medium. By a conformal mapping method, we can try to calculate these values and so, the capacitances would only depend on ε_o, ε_r, w/h and s/h.

This result is very important because it makes unusable very long and expansive computing calculations to obtain impedances (odd and even mode) coupling coefficient, adaptation, etc... for microstrip couplers, phase displacement or group delay for meander lines for example.

References

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2. R. Daumas, D. Pompei, E. Rivier, A. Ros, "Faster impedance estimation for coupled microstrips with an overrelaxation method." IEEE Trans. on MTT, August 1973, pp. 552-556.

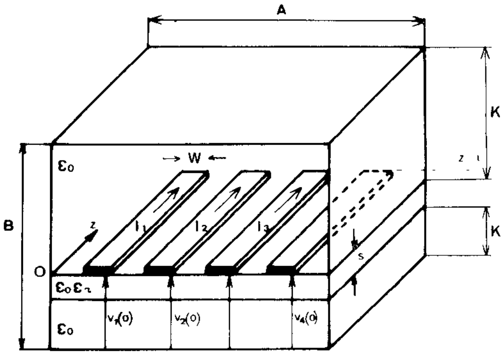

Fig. 1 Geometrical arrangement of a n multilines coupler.